Mathematical Foundations of Supersymmetry

Satyanarayan Ray Pitambar Mohapatra
Project Guide : Prof. Probir Roy, TIFR
Department of Theoritical Physics
VSRP project report

August 19, 2003

Abstract

The proofs of two fundamental theorems, on which the formulation of supersymmetry rests, are examined in detail. The first one, due to Coleman and Mandula, forbids the fusion of bosonic internal symmetries with Poincaré invariance in any way other than the trivial. The second one is the Hagg-Łopunszánsky-Sohnius theorem which specifies the most general structure of the fermionic symmetry that is allowed to be fused nontrivially with the symmetry of the Lorentz group.

1 Introduction

The Coleman and Mandula theorem puts a straightjacket on the type of symmetry the S-matrix can have. This theorem makes it impossible to fit internal and spacetime symmetry groups into a bigger group, where the bigger group is not just a product of former and the Poincaré group. In the derivation of the Coleman and Mandula theorem the internal symmetry generators are assumed to be bosonic, obeying a Lie algebra. But by taking into account anticommutation and extending the Lie algebra to a Z_2 graded Lie algebra, one can have a nontrivial fusion of the fermionic symmetry with the Lorentz group. A consequence of the HLS theorem is the most general form of the algebra of supersymmetry which leads to supermultiplets with bosonic and fermionic partners.

2 Coleman and Mandula Theorem

Theorem 1 (Coleman and Mandula) *The full symmetry group G of the S-matrix, is locally isomorphic to the direct product of an internal symmetry group and the Poincaré group.*

2.0.1 Assumptions

1. "Particle number finiteness": For any mass M there are only a finite number of particle types with mass less than M.
2. "Occurence of scattering": Any two-paricle state undergoes some scattering at almost all energies (i.e. except possibly an isolated set).
3. "Weak-elastic analyticity": The amplitude for elastic 2-body scattering is an analytic function of the scattering angle at almost all energies and angles.
4. The generators of the symmetry group of the S-matrix are integral operators in momentum space whose kernels are distributions.

2.1 Proof

A symmetry generator has the following properties:

- It is Hermitian.
- It commutes with the S-matrix.
- The commutator of two symmetry generators is also a symmetry generator.
- It takes single particle states on the mass-shell into single particles states of the same mass (O'Raifairteigh's theorem).
- Its action on a multiparticle state is the direct sum of its action on individual single particle components.

2.1.1 Case-I

Here we take the internal symmetry generators to commute with the four momentum operator P_μ:

$$[B_\alpha, P_\mu] = 0.$$

The Lie algebra structure of the group is

$$[B_\alpha, B_\beta] = i \sum_\gamma C^\gamma_{\alpha\beta} B_\gamma.$$

where $C^\gamma_{\alpha,\beta}$, are the structure constants. The action of B_α on a one particle state is

$$B_\alpha \mid p\rangle = b_\alpha(p) \mid p\rangle.$$

In general,

$$B_\alpha \mid pm\rangle = \sum_{m'} b_\alpha(p)_{m'm} \mid pm'\rangle.$$

where m denotes both the spin index and particle type of definite mass $\sqrt{-p_\mu p^\mu}$. The mapping from B_α to $b_\alpha(p)$ is a *homomorphism* but this is not an *isomorphism*. Isomorphism would require

$$\sum_\alpha c^\alpha b_\alpha(k) = 0 \ \forall \ momenta \ \ k \ \ .$$

so that $\sum_\alpha c^\alpha B_\alpha = 0$. *But* $\sum_\alpha c^\alpha b_\alpha = 0$ for a particular p does not imply

$$\sum_\alpha c^\alpha b_\alpha(k) = 0 \ \ (\forall \ \ k).$$

In a two particle state a particle (of momentum and type) p can elastically scatter with a particle q to give particles p' and q', where the masses are

$$\sqrt{-p'_\mu p^{\mu\prime}} = \sqrt{-p_\mu p^\mu} \ \ , \ \ \sqrt{-q'_\mu p^{\mu\prime}} = \sqrt{-q_\mu q^\mu}.$$

We then have

$$B_\alpha \mid p, q\rangle = b_\alpha(p,q) \mid p, q\rangle,$$

where

$$(b_\alpha(p,q))_{m'n',mn} = ((b_\alpha(p))_{m'm}\delta_{n'n} + (b_\alpha(q))_{n'n}\delta_{m'm}$$

and $\delta_{mm} \ = N(\sqrt{-p_\mu p^\mu})$, the multiplicity of p-type particles etc. The invariance of the S-matrix implies the similarity transformation,

$$b_\alpha(p',q')S(p',q';p,q) = S(p',q';p,q)b_\alpha(p,q).$$

Taking the trace,we have

$$Tr \ b_\alpha(p',q') = Tr \ b_\alpha(p,q),$$

i.e.

$$N(\sqrt{-q_\mu q^\mu}) \ \ trb_\alpha(p') + N(\sqrt{-p_\mu p^\mu}) \ \ trb_\alpha(q') = N(\sqrt{-q_\mu q^\mu}) \ \ trb_\alpha(p) + N(\sqrt{-p_\mu p^\mu}) \ \ trb_\alpha(q).$$

Using the finiteness of N, one can show that $b_\alpha(p)$ are finite hermitian matrices. The conservation of momentum ($p' + q' = p + q$) and the above equation can be used to show that the function

$$\frac{tr \ \ b_\alpha}{N(\sqrt{-p_\mu p^\mu})}$$

is linear in p_μ.

Define a new operator as

$$B^\#_\alpha \equiv B_\alpha - a^\mu_\alpha P_\mu,$$

where

$$B^\#_\alpha \mid pm\rangle = b^\#_\alpha(p)_{m'm} \mid pm'\rangle.$$

One can show that $B^\#_\alpha$ obeys the original Lie algebra and commutes with the S-matrix and furthermore

$$\sum_\alpha c^\alpha b^\#_\alpha(k) = 0 \ \ \forall \, k.$$

Thus an *isomorphism* gets established between $B_\alpha^\#$ and $b_\alpha^\#(p)$. Now a theorem[1] of Lie Algebras immediately gives the result that the set $\{B_\alpha^\#\}$ must be the direct sum of a compact semi-simple Lie algebra and $U(1)$ generators. By suitably choosing a Lorentz generator J, which leaves any pair of mass-shell momenta p and q invariant, one can dispose of the U(1) Lie algebras. It can be further shown that the generators of the semi-simple compact part of the the Lie algebra, say $B_\beta^\#$, commute with Lorentz transformation and they are the genarators of internal symmetries. So we get, the symmetry genarators B_α that commute with P_μ are either internal symmetry generators or linear combinations of the componentx of P_μ itself. This is the proof of Coleman-Mandula theorem when the internal symmetry generators commute with the four momenta operator.

2.1.2 Case-II

Here we have the symmetry generators A_α not commuting with P_α:

$$[A_\alpha, P_\mu] \neq 0.$$

We can write

$$A_\alpha \mid p, n\rangle = \sum_{n'} \int d^4p' (\mathcal{A}_\alpha(p', p))_{n'n} \mid p', n'\rangle,$$

where the kernel $\mathcal{A}_\alpha$ is given b[2]y

$$\mathcal{A}_\alpha(p' - p) = \sum_{n=0}^{D_\alpha} C_n {\delta_n}^{(4)}(p' - p).$$

$\delta_n^{(4)}$ being an n^{th} momentum derivative of the four dimensional delta-function in momentum space and D_α is the order of the highest derivative present in the sum.

Define

$$B_\alpha^{\mu_1 \ldots \mu_{D_\alpha}} \equiv [P^{\mu_1}, [P^{\mu_2}, \ldots [P^{\mu_{D_\alpha}}, A_\alpha]] \ldots]$$

Since $D_\alpha + 1$ factors of $(p' - p)$, multiplied by D_α momentum derivatives on $\delta^{(4)}(p - p')$, vanish, one can get the following.

$$\langle p \mid [B_\alpha^{\mu_1 \ldots \mu_{D_\alpha}}, P^\mu] \mid p'\rangle = 0.$$

If

$$B_\alpha^{\mu_1 \ldots \mu_{D_\alpha}} \mid p\rangle = b_\alpha^{\mu_1 \ldots \mu_{D_\alpha}}(p) \mid p\rangle,$$

one can write

$$b_\alpha^{\mu_1 \ldots \mu_{D_\alpha}}(p) = b_\alpha^{\sharp\mu \ldots \mu_{D_\alpha}} + a_\alpha^{\mu\mu_1 \ldots \mu_{D_\alpha}} p_\mu.1.$$

[1]A Lie algebra of finite Hermitian matrices is at most the direct sum of a semi-simple compact Lie algebra and some number of $U(1)$ Lie algebras.

[2]This means the nth derivative of a delta function. The fourth assumption on page-2 implies that the kernel of a generator is a distribution in momentum space. Then a theorem from distribution theory is used. It says that a distribution which vanishes everywhere except at one point can be expanded as a finite sum of derivatives of delta functions.

Hence $b_\alpha^{\sharp\mu_1...\mu_{D_\alpha}}$ is traceless, and both $b_\alpha^{\sharp\mu_1...\mu_{D_\alpha}}$ and $a_\alpha^{\sharp\mu\mu_1...\mu_{D_\alpha}}$ are momentum independent. By O'Raifairteigh's theorem,

$$[A_\alpha, -P_\mu P^\mu] = 0.$$

For $D_\alpha \geq 1$, we have

$$0 = [P^{\mu_1}P_{\mu_1}, [P^{\mu_2}, ...[P^{\mu_{D_\alpha}}, A_\alpha]]...] = [P^{\mu_1}, ...[P^{\mu_{D_\alpha}}, A_\alpha]]...]P_{\mu_1} + P^{\mu_1}[P_{\mu_1}, ...A_\alpha]]...] = 2P_{\mu_1}B_\alpha^{\mu_1...\mu_{D_\alpha}}$$

and therefore

$$p_{\mu_1}b_\alpha^{\mu_1...\mu_{D_\alpha}}(p) = 0.$$

For a massive particle

$$p^\mu p_\mu \neq 0$$

and the above must hold for p in any timelike direction. So

$$b_\alpha^{\sharp\mu...\mu_{D_\alpha}}(p) = 0$$

and

$$a_\alpha^{\mu\mu_1\mu_2...\mu_{D_\alpha}} = -a_\alpha^{\mu_1\mu\mu_2...\mu_{D_\alpha}}.$$

But

$$a_\alpha^{\mu\mu_1\mu_2...\mu_{D_\alpha}}$$

is symmetric under exchange of indices. So there are two options.
Either

$$D_\alpha = 0 \ \ or \ \ D_\alpha = 1,$$

since, for $D_\alpha \geq 2$,

$$a_\alpha^{\mu\mu_1\mu_2...\mu_{D_\alpha}} = 0,$$

which is a trivial case. For $D_\alpha = 0$

$$[A_\alpha, P^\mu] = 0.$$

This case has already been discussed in Case-I

For $D_\alpha = 1$, one has $b_\alpha^{\#\mu} = 0$.
So

$$[A_\alpha, P^\mu] = a_\alpha^{\mu\nu}P_\nu.$$

Thus the general form of A_α is

$$A_\alpha = -\frac{1}{2}ia_\alpha^{\mu\nu}J_{\mu\nu} + B_\alpha, where$$

$J_{\mu\nu}$ are the generators of the homogenous Lorentz group with

$$[P_\nu, J_{\rho\sigma}] = -i_{\nu\rho}P_\sigma + i_{\nu\sigma}P_\rho$$

and B_α are symmetry generators that commute with P_μ.
Finally, since B_α commute with P_α, in general they are linear combinations of internal symmetry generators and components of P_μ. This completes the proof of the Coleman-Mandula theorem.

3 HLS theorem

Theorem 2 (Hagg,Łopuszánski,Sohnius) *The most general continuous symmetry of the S-matrix, consistent with the assumtions of the Coleman-Mandula theorem, is that pertaining to a Z_2 - graded Lie algebra where the even operators are a direct sum of the Poincaré and the other symmetry generators (i.e the latter two sets of generators mutually commute). The Z_2-odd operators belong to the representations $(0, \frac{1}{2})$ and $(\frac{1}{2}, 0)$ of the homogeneous Lorentz group.*

3.1 Discussion

- Q_{ab}^{AB} are a set of operators forming an (A,B) representation of the homogeneous Lorentz group.
- A, B are either integers or half-integers.
- a,b are indices that run from -A to +A and -B to +B respectively. So there are (2A+1)(2B+1) number of operators.
- $\mathbf{A} = \frac{1}{2}(\mathbf{J} + i\mathbf{K})$ and $\mathbf{B} = \frac{1}{2}(\mathbf{J} - i\mathbf{K})$. $\mathbf{J}$ are the generators of rotations, $\mathbf{K}$ are the generators of boosts.
- They satisfy the following commutation relations.

$$[\mathbf{A}, Q_{ab}^{AB}] = -\sum_{a'} \mathbf{J}_{aa'}^{(A)} Q_{a'b}^{AB} \qquad and \qquad [\mathbf{B}, Q_{ab}^{AB}] = -\sum_{a'} \mathbf{J}_{bb'}^{(B)} Q_{ab'}^{AB}.$$

So effectively the following lemmas have to be proved.

1. The anticommutation of fermionic generators and their adjoints involve nonzero bosonic symmetry generator belonging to the $(A+B, A+B)$ representation.
2. The fermionic generators can only belong to the $(0, \frac{1}{2})$ *and* $(\frac{1}{2}, 0)$ representation.
3. The fermionic generators satisfy the following anticommuatation relation with their adjoints[3].

$$\{Q_{ar}, Q_{bs}^*\} = 2\delta_{rs}\sigma_{ab}^{\mu}P_{\mu}.$$

4. The fermionic generators commute with momentum.

$$[P_\mu, Q_{ar}] = [P_\mu, Q_{ar}^*] = 0.$$

5. The fermionic generators obey the following anticommutation relation with each other giving rise to internal bosonic symmetry generators Z_{rs} known as central charges.

$$\{Q_{ar}, Q_{bs}\} = e_{ab}Z_{rs}.$$

   Where e_{ab} are some Clebsch-Gordon coefficients.

6.

$$[Z_{rs}, Q_{at}] = [Z_{rs}, Q_{at}^*] = [Z_{rs}, Z_{tu}] = [Z_{rs}, Z_{tu}^*] = [Z_{rs}^*, Q_{at}] = [Z_{rs}^*, Q_{at}^*] = [Z_{rs}^*, Z_{tu}^*] = 0.$$

[3]Here * means hermitian adjoint

3.2 Proof

3.2.1 Proof of lemma 1

If Q_{ab}^{AB} belongs to (A,B) representation. Then Q_{ab}^{AB*} belongs to (B,A) representation, since

$$Q_{ab}^{AB*} = (-1)^{A-a}(-1)^{B-b}\bar{Q}_{-b-a}^{BA},$$

which is a similarity transformation and $\bar{Q}_{ba}^{BA}$ transform according to the representation (B,A). Now

$$\{Q_{ab}^{AB}, Q_{ab}^{AB*}\} = (-1)^{A-a'}(-1)^{B-b'} \sum_{C=|A-B|}^{A+B} \sum_{D=|A-B|}^{A+B} \sum_{c=-C}^{C} \sum_{d=-D}^{D} C_{AB}(Cc;a-b')C_{AB}(Dd;-a'b)X_{cd}^{CD}.$$

It follows that

$$X_{A+B,-A-B}^{A+B,A+B} = (-1)^{2B}\{Q_{A,-B}^{A,-B}, Q_{A,-B}^{AB*}\},$$

where X_{cd}^{CD} is the (c,d)-component of an operator that tranforms according to (C,D) representation of the homogeneous Lorentz group. C_{AB} are the Clebsch-Gordon coefficients for coupling A and B.

$$X_{A+B,-A-B}^{A+B,A+B} = 0 \qquad if \qquad Q_{A,-B}^{AB} = 0.$$

But then, by using raising and lowering operators, one sees that all $Q_{ab}^{AB} = 0$. So, for any nonvanishing fermionic generator, its anticommutation with its adjoint belongs to the $(A+B, A+B)$ representation. (Proved.)

3.2.2 Proof of $(0,\frac{1}{2})$ *and* $(\frac{1}{2},0)$ representation (lemma 2)

A symmetric traceless tensors of rank N transforms acoording to the representation $(\frac{N}{2},\frac{N}{2})$. An antisymmetric tensor of rank 2 transforms according to the representation $(1,0)\oplus(0,1)$. The Dirac field tranforms according to the representation $(\frac{1}{2},0)\oplus(0,\frac{1}{2})$.

According to the Coleman-Mandula theorem the bosonic symmetry generators consist of P_μ(tranlations genarators, representation $(\frac{1}{2},\frac{1}{2})$), $J_{\mu\nu}$(Lorentz transformation, representation $(1,0)\oplus(0,1)$) and T_A (internal bosonic symmetry,representation $(0,0)$). But it is shown in the above subsection that the nonzero bosonic operator arising from the anticommutation of a fermionic operator and its conjugate belongs to the $(A+B, A+B)$ representation. As the bosonic symmetry generator consist of $(\frac{1}{2},\frac{1}{2})$, the fermionic generator would belong to (A,B) such that

$$(A+B) \leq \frac{1}{2}$$

Thus fermionic symmetry generators can only belong to $(0,\frac{1}{2})$ *and* $(\frac{1}{2},0)$. (Proved.)

3.2.3 Proof of lemma 3

Q_{ar} belongs to $(0,\frac{1}{2})$ representation Q_{bs}^* belongs to $(\frac{1}{2},0)$ representation. So $\{Q_{ar}, Q_{bs}^*\}$ must belong to $(\frac{1}{2},\frac{1}{2})$ representation. As

$$(0,\frac{1}{2})\otimes(\frac{1}{2},0) = (\frac{1}{2},\frac{1}{2})$$

Using the completeness of σ_μ matrices, we have

$$\{Q_{bs}, Q^*_{ar}\} = N^\mu_{rs}(\sigma_{\mu ab}),$$

where N^μ is a matrix operator. It has the transformation property

$$U^{-1}(\Lambda)N^\mu U(\Lambda) = \Lambda^\mu_\nu N^\nu.$$

So this is a four vector operator. The Coleman-Mandula theorem asserts that P^μ is the only four vector bosonic symmetry operator. So $N^\mu \propto P^\mu$ or $N^\mu_{rs} = 2P^\mu N_{rs}$.

As

$$\{Q_{ar}, Q^*_{bs}\}^* = \{Q_{bs}, Q^*_{ar}\} = 2N^*_{rs}\sigma^\mu_{ab}P_\mu = 2N_{sr}\sigma^\mu_{ab}P_\mu,$$

N_{rs} is Hermitian.

Q_{ar} are linearly independent, so

$$Q \equiv \sum_r d_a c_r Q_{ar} \quad \neq 0$$

except for trivial case where all the coefficients vanish. Now

$$\langle \Psi \mid \{Q, Q^*\} \mid \Psi\rangle > 0,$$

or

$$\langle \Psi \mid \sum_{ab}\sigma^\mu_{ab}P_\mu d_a d^*_b \mid \Psi\rangle \sum_{rs} c_r c^*_s N_{rs} > 0.$$

This means that

$$\sum_{rs} c_r c^*_s N_{rs} > 0,$$

except for the trivial case where all c's or d's are zero. But $\sum_{ab}\sigma^\mu_{ab}P_\mu d_a d^*_b$ acting on any on-mass-shell state is positive definite. This guarantees that N_{rs} is positive definite.

Defining

$$Q'_{ar} \equiv \sum_s N_{rs}^{-\frac{1}{2}} Q_{as},$$

one is led to

$$\{Q'_{ar}, Q'^*_{bs}\} = 2\delta_{rs}\sigma^\mu_{ab}P_\mu,$$

where $N_{rs}^{-\frac{1}{2}}$ is the normalization constant. (Proved.)

3.2.4 Proof of lemma 4

P_μ is an operator represented as $(\frac{1}{2}, \frac{1}{2})$,

Q_{ar} is an operator represented as $(0, \frac{1}{2})$.

So $[Q_{ar}, P_\mu]$ is an operator represented as either

$$(\frac{1}{2} + 0, \frac{1}{2} + \frac{1}{2}) \ \ or \ \ (\frac{1}{2} - 0, \frac{1}{2} - \frac{1}{2}),$$

i.e. either

$$(\frac{1}{2},1)\ \ or\ \ (\frac{1}{2},0).$$

But a symmetry generator like

$$(\frac{1}{2},1)$$

does not exist, according to lemma 1. Hence

$$[Q_{ar},P_\mu]\propto Q^*,$$

which is the $(\frac{1}{2},0)$ representation.

Define

$$\mathcal{M}\equiv\sigma_\mu P^\mu.$$

Now,

$$[\mathcal{M}_{ab},Q_{cr}]=\sum_s e_{ac}K_{rs}Q^*_{bs}$$

where e_{ac} is the Clebsch-Gordan coefficient. For spin the singlet case the above equation would give

$$[\mathcal{M}_{-\frac{1}{2},-\frac{1}{2}},[\mathcal{M}_{-\frac{1}{2},-\frac{1}{2}},Q_{\frac{1}{2}r},Q^*_{\frac{1}{2}s}]]=-4\mathcal{M}_{-\frac{1}{2},-\frac{1}{2}}KK^\dagger{}_{rs}.$$

Now the left hand side can be written as

$$[\mathcal{M}_{-\frac{1}{2},-\frac{1}{2}},[\mathcal{M}_{-\frac{1}{2},-\frac{1}{2}},2\delta_{rs}\sigma^\mu{}_{-\frac{1}{2},-\frac{1}{2}}P_\mu]]=[(\sigma_\mu P^\mu)_{-\frac{1}{2},-\frac{1}{2}},[(\sigma_\mu P^\mu{}_{-\frac{1}{2},-\frac{1}{2}},2\delta_{rs}\sigma^\mu{}_{-\frac{1}{2},-\frac{1}{2}}P_\mu]]$$

$= 0.$

Since $\mathcal{M}_{-\frac{1}{2},-\frac{1}{2}}\neq 0$ for nontrivial momenta,

$$KK^\dagger=0\ \ which\ \ implies\ \ K=0,$$

leading to

$$[Q_{ar},P_\mu]=0.$$

By complex conjugation one can show

$$[Q^*_{ar},P_\mu]=0.$$

(Proved.)

3.2.5 Proof of lemma 5

$\{Q_{ar},Q_{bs}\}$ would belong to the linear combination of

$$(0+0,\frac{1}{2}+\frac{1}{2})\ \ and\ \ (0+0,\frac{1}{2}-\frac{1}{2}),$$

i.e. the representations in

$$(0,0)\ \ and\ \ (0,1).$$

Since

$$[Q_{ar}, P_\mu] = 0,$$

one gets

$$[\{Q_{ar}, Q_{bs}\}, P_\mu] = 0.$$

Also,

$$[P_\mu, J_{\mu\nu}] \neq 0$$

implies that

$$\{Q_{ar}, Q_{bs}\}$$

is not a $(0,1)$ symmetry generator. Rather, it leads to a $(0,0)$ symmetry generator. i.e. it is an internal symmetry generator.

$$\{Q_{ar}, Q_{bs}\} = e_{ab} Z_{rs}$$

The left hand side is a symmetric tensor. As e_{ab} is an antisymmetric tensor, Z_{rs} should be an antisymmetric tensor. i.e.

$$Z_{rs} = -Z_{sr}$$

(Proved.)

3.2.6 Proof of lemma 6

Jacobi identity for Q_{ar}, Q_{bs}, Q^*_{bs} is

$$[\{Q_{ar}, Q_{bs}\}, Q^*_{bs}] + [\{Q_{bs}, Q^*_{ct}\}, Q_{bs}] + [\{Q^*_{ct}, Q_{ar}\}, Q_{bs}] = 0.$$

The last two terms give zero. So

$$[\{Q_{ar}, Q_{bs}\}, Q^*_{bs}] = 0.$$

From another Jacobi identity

$$-\{Z_{rs}, Q_{at}, Q^*_{bu}] + [\{Q^*_{bu}, [Z^*_{rs}\}, Q_{at}] - \{Q_{at}, [Q^*_{bu}\}, Z_{rs}] = 0,$$

one gets

$$\{Q^*_{bu}, [Z^*_{rs}, Q_{at}]\} = 0.$$

Now $[\{Q^*_{bu}, Z^*_{rs}\}, Q_{at}]$ belongs to $(0, \frac{1}{2})$. So

$$[Z^*_{rs}, Q_{at}] = 0.$$

Complex conjugation would give

$$[Z_{rs}, Q^*_{ct}] = 0.$$

Other properties of the central charges can be derived similarly. (Proved.)

4 Discussion/Conclusion

The Coleman-Mandula theorem disallows any nontrivial fusion between bosonic symmetries and Poincaré invariance. The HLS theorem shows that fermionic symmetries fusing non-trivially with Lorentx invariance, are allowed for the S-matrix. These fermionic generators can be taken in the $(\frac{1}{2}, 0) \oplus (0, \frac{1}{2})$ representation of the homogeneous Lorentz group. They commute with P_μ but not with $J_{\mu\nu}$. Their anticommutators are hermitian bosonic generators called central charges which obey the Coleman-Mandula theorem. The algebra of these fermionic generators is the supersymmetry algebra. It leads to supermultiplets with fermion and boson partners, but we have not gone into that aspect.

5 Acknowledgments

I am grateful to my project guide Prof. Probir Roy for his guidance and help.

6 References

1. S. Coleman and J. Mandula, *Phys. Rev.* **159**, 1251 (1967).

2. R. Hagg, J.T. Łopunszánsky and M.Sohnius, *Nucl. Phys.* **B88**, 257 (1975).

3. M.Drees, R.M.Godbole and Probir Roy, *Theory and Phenomenology of Sparticles* (World Scientific).

4. H.J.W. Müller-Kirsten, A.Wiedemann, *Supersymmetry: An Introduction with Conceptual and Calculational Details*(World Scientific, 1987).

5. Steven Weinberg, *The Quantum theory of Fields*, Volume III Supersymmetry (Cambridge University press, 2000).

6. Yu.V. Novozhilov, *Introduction to elementary particle theory*, (Pergamon Press, 1975).

7. Michale E. Peskin and Daniel V. Schroeder *An Introduction to Quantum Field Theory* (Addison Wesley Publishing Company, 1995).